Roll No. _______ **Code No. Q-4312**

SECOND SEMESTER EXAMINATION 2021-22

M.Sc. Mathematics

Paper - II

Real Analysis - II

Time : 3.00 Hrs. **Max. Marks : 80**

Total No. of Printed Page : 03 **Mini. Marks : 29**

Note: Question paper is divided into three sections. Attempt question of all three section as per direction. Distribution of Marks is given in each section.

Section - 'A'

Very short type question (in few words). **6x2=12**

Q.1 Attempt any six question from the following questions :

(i) Write translation invariant.

(ii) Write outer measure of cantor set.

(iii) Define characteristic function.

(iv) Define Borel set.

(v) State Bounded convergence theorem.

(vi) If f is bounded measurable function defined on a set E and $m(E)=0$ Then write the value of $\int_E f$.

(vii) A measurable function of continuous. It is True of false.

(viii) "A bounded function is necessarily a function of bounded variation " Whether this statement is 'true' or 'false'.

(ix) State Holder's inequality.

(x) Define L^{∞}-space.

Section - 'B'

Short answer question (In 200 words) **4x5=20**

Q.2 Attempt any four question from the following questions :

(i) Let $\{E_n\}$ be a countable collection of sets of real numbers. Then

$$m^{\forall}\left(\bigcup_{n=1}^{\infty} E_n\right) \leq \sum_{n=1}^{\infty} m^{*}(E_n)$$

(ii) Prove that the interval (a,∞) is measurable.

(iii) Prove that a continous function defined on a measurable set is measurable.

(iv) Show that a function is simple if and only if it is measurable and assumes only a finite number of values.

(v) If $\int_E f = 0$ and $f(x) \geq 0$ on E then prove that

$f = 0$ *a.e. on* E

(vi) Using lebesgue dominated convergence theorem Evaluate following :

$$\lim_{n\to\infty} \int_0^1 \frac{n^{3/2}x}{1+n^2x^2}\,dx,\ x \in \{0,1\}, n = 1,2,3.....$$

(vii) Prove that every absolutely continuous function is of bounded variation.

(viii) If $f, g \in L^p[a,b]$. Then prove that $f + g \in L^p[a,b]$.

Section - 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four question from the following questions :

(i) Prove that outer measure of an interval is its length.

(ii) Prove that union of measurable set is a measurable set.

(iii) Let f and g be measurable function on E. Then prove that each of the following functions is measurable on E :

(a) $f+g$ (b) $f-g$ (c) fg (d) $\frac{f}{g}(g(x) \neq 0 \text{ on } E)$

(iv) (a) Let f be a function defined on a measurable set E. Prove that f is measurable if and only if, for any open set G in R the inverse image $f^{-1}(G)$ is a measurable set.

(b) Let f and g be two function defined on a common domain E such that $f = g$ $a.e.$ and g is measurable. Prove that f is measurable on E ·

(v) (a) State and prove monotone convergence theorem.

(b) State and prove Frechet theorem.

(vi) (a) Let f and g be non-negative measurable function defined on a set E. Then prove that :

$$\int_E (f+g) = \int_E f + \int_E g$$

(b) State and prove Fatou's lemma.

(vii) (a) If f is of bounded variation on $[a,b]$ then

$T_a^b = P_a^b + N_a^b$ and $f(b) - f(a) = P_a^b - N_a^b$

(b) Prove that L^P space is complete.

(viii) State and prove Schwarz's Inequality.

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